26. Sketch the curve $y = \frac{1}{x-3}$.
[2077]

Solution:

here, $f(x) = \frac{1}{x-3}$

i. Domain:

here, for domain to exist $x - 3 \neq 0 \Rightarrow x \neq 3$.

$\therefore$ Domain $= (-\infty, 3) \cup (3, \infty)$

ii. Interceps:

put $x = 0$, $y = -1/3$

put $y = 0$, $x = \infty$

Thus, the curve meets the x-axis and y-axis at origin.

iii. ~~Symmetricity~~ Asymptotes:

Horizontal asymptotes;

$$\lim_{x \to \infty} \frac{1}{x-3} = \lim_{x \to \infty} \frac{1/x}{1 - 3/x} = 0.$$

$\therefore y = 0$ is horizontal asymptote.

Vertical asymptotes:

$$\lim_{x \to a} f(x) = \infty \Rightarrow \lim_{x \to 3} \frac{1}{x-3} = \infty$$

here, $f(x)$ is ∞ at, $x - 3 = 0 \Rightarrow x = 3$.

$\therefore x = 3$ is a vertical asymptote.

iv. Symmetricity:

put $x = -x$,

$$f(-x) = \frac{1}{-x-3}$$

So, no symmetrical.

v. Increasing and decreasing:

$$f'(x) = \frac{-1}{(x-3)^2}.$$

At critical point,

$$\frac{-1}{(x-3)^2} = 0.$$

$$(-1 = 0) \text{ false}.$$

So,

$$f'(x) \to \infty.$$

or, $$\frac{-1}{(x-3)^2} = \frac{1}{0}$$

or, $$(x-3)^2 = 0$$

$$\therefore x = 3.$$

Interval	Sign of $f'(x)$	Nature of $f(x)$
$(-\infty, 3)$	-ve	decreasing
$(3, \infty)$	-ve	decreasing

vi. Maxima and minima:

Since, $f(x)$ is decreasing.

Here, no maxima and minima.

vii. Concavity:

$$f''(x) = \frac{2}{(x-3)^3}$$

At point of inflection,

$$f''(x) = 0$$

or, $\frac{2}{(x-3)^3} = 0$

$2 = 0$ (false)

Also,

$$f''(x) \to \infty$$

or, $\frac{2}{(x-3)^3} = \frac{1}{0}$

or, $(x-3)^3 = 0$

$\therefore x = 3$

Interval	Sign of $f''(x)$	Nature of $f(x)$
$(-\infty, 3)$	$-$ve	concave downward
$(3, \infty)$	$+$ve	concave upward

Using above information to sketch the graph,